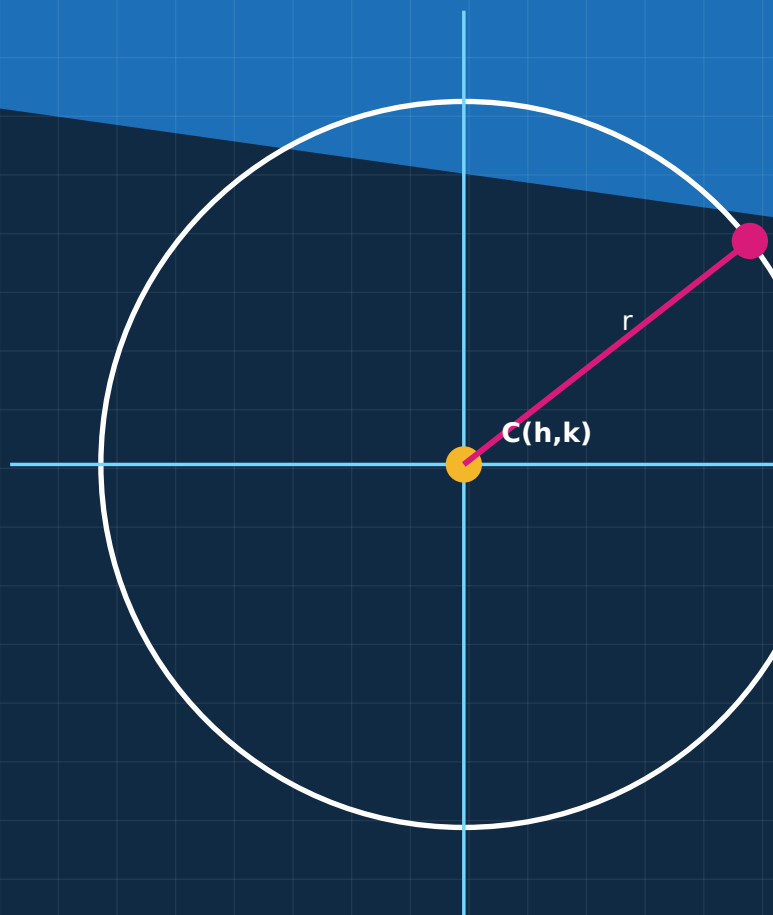


CIRCLES

FORMULA & QUICK REVISION SHEET



$$(x - h)^2 + (y - k)^2 = r^2$$

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

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1. Standard Forms & Basic Circle Results

JEE Main • High-yield formulae, definitions and quick-use results

Standard Equation

- $(x - h)^2 + (y - k)^2 = a^2 \rightarrow$ centre (h, k) , radius a .
- Centre at origin: $x^2 + y^2 = a^2$.
- Circle through origin: $x^2 + y^2 - 2hx - 2ky = 0$.
- Circle touching X-axis: $x^2 + y^2 - 2hx - 2ay + h^2 = 0$.
- Circle touching Y-axis: $x^2 + y^2 - 2ax - 2ky + k^2 = 0$.
- Circle touching both axes: $x^2 + y^2 - 2ax - 2ay + a^2 = 0$.

Parametric Form

- For $(x-h)^2 + (y-k)^2 = a^2$: $x = h + a \cos\theta$, $y = k + a \sin\theta$, $0 \leq \theta \leq 2\pi$.
- For $x^2 + y^2 = a^2$: $x = a \cos\theta$, $y = a \sin\theta$.

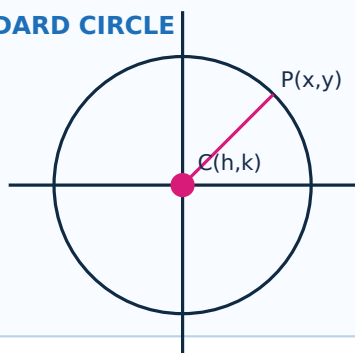
Special Cases

- Passing through origin with centre on X-axis: $x^2 + y^2 - 2ax = 0$.
- Passing through origin with centre on Y-axis: $x^2 + y^2 - 2ay = 0$.
- Passing through origin and cutting intercepts a, b on axes: $x^2 + y^2 - 2ax - 2by = 0$.
- Diameter endpoints $(x_1, y_1), (x_2, y_2)$: $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$.

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$(x-h)^2 + (y-k)^2 = a^2$	$x = h + a \cos\theta, y = k + a \sin\theta$	Diameter form: $(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$
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STANDARD CIRCLE



$$(x-h)^2 + (y-k)^2 = r^2$$

centre + radius

2. General Equation, Position of Point & Tangent

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General Equation

- $x^2 + y^2 + 2gx + 2fy + c = 0$.
- Centre = $(-g, -f)$.
- Radius = $\sqrt{g^2 + f^2 - c}$.
- $g^2 + f^2 - c > 0 \rightarrow$ real circle; $= 0 \rightarrow$ point circle; $< 0 \rightarrow$ imaginary circle.

Position of a Point

- For $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, calculate $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$.
- $S_1 > 0 \rightarrow$ outside; $S_1 = 0 \rightarrow$ on circle; $S_1 < 0 \rightarrow$ inside.

Intercepts on Coordinate Axes

- Length of X-axis intercept = $2\sqrt{g^2 - c}$, when real.
- Length of Y-axis intercept = $2\sqrt{f^2 - c}$, when real.
- On X-axis: $g^2 > c \rightarrow$ two intersections; $g^2 = c \rightarrow$ tangent; $g^2 < c \rightarrow$ no real intersection. Similarly use f^2 versus c for Y-axis.

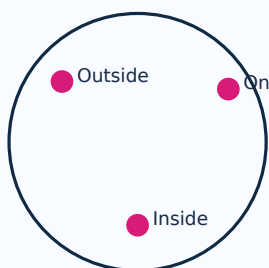
Tangent — Point Form

- For $x^2 + y^2 + 2gx + 2fy + c = 0$ at $P(x_1, y_1)$: $xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0$.
- For $x^2 + y^2 = r^2$ at $P(x_1, y_1)$: $xx_1 + yy_1 = r^2$.

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C = $(-g, -f)$	R = $\sqrt{g^2 + f^2 - c}$	Tangent at P: T = 0
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POSITION OF A POINT



$$S > 0 / S = 0 / S < 0$$

outside • on • inside

3. Tangent, Normal & Key Results

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Tangent — Slope / Parametric Forms

- For $x^2+y^2+2gx+2fy+c=0$, tangent of slope m : $y+f = m(x+g) \pm \sqrt{[(g^2+f^2-c)(1+m^2)]}$.
- For $(x-a)^2+(y-b)^2=r^2$: $y-b = m(x-a) \pm r\sqrt{1+m^2}$.
- For $x^2+y^2=r^2$: $y = mx \pm r\sqrt{1+m^2}$.
- At point $(a+r \cos\theta, b+r \sin\theta)$: $(x-a)\cos\theta + (y-b)\sin\theta = r$.

Normal

- For $x^2+y^2+2gx+2fy+c=0$ at (x_1, y_1) : $(y+f)(x_1+g) = (x+g)(y_1+f)$.
- For $x^2+y^2=r^2$ at (x_1, y_1) : $x/x_1 = y/y_1$.
- Parametric point $(r \cos\theta, r \sin\theta)$: $y = x \tan\theta$.

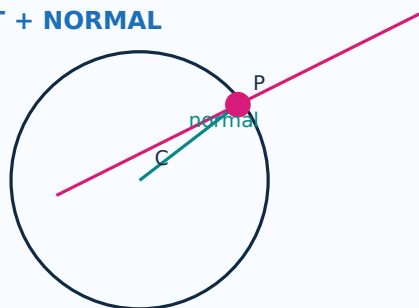
High-Value Results

- Other end of diameter when one end is (x_1, y_1) for $x^2+y^2+2gx+2fy+c=0$: $(-2g-x_1, -2f-y_1)$.
- Line $lx+my+n=0$ touches $x^2+y^2=r^2$ if $n^2 = r^2(l^2+m^2)$.
- A line cuts a circle at two distinct real points when perpendicular distance from centre $<$ radius.
- For $y=mx+c$ and $x^2+y^2=a^2$, chord length $= 2\sqrt{[a^2(1+m^2)-c^2]}/\sqrt{1+m^2}$.
- Maximum and minimum distances of point P from a circle of centre C and radius r are $CP+r$ and $|CP-r|$.

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$y+f=m(x+g) \pm \sqrt{[(g^2+f^2-c)(1+m^2)]}$	Normal passes through centre	Line $lx+my+n=0$ tangent $\Leftrightarrow n^2=r^2(l^2+m^2)$
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TANGENT + NORMAL



Tangent $\perp$ radius

normal passes through centre

4. Pair of Tangents, Chord of Contact & Polar

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Pair of Tangents

- From $P(x_1, y_1)$ to $x^2 + y^2 = r^2$:
 $(x^2 + y^2 - r^2)(x_1^2 + y_1^2 - r^2) = (xx_1 + yy_1 - r^2)^2$, equivalently $S S_1 = T^2$.
- Length of tangents from $P(x_1, y_1)$ to $x^2 + y^2 + 2gx + 2fy + c = 0 = \sqrt{S_1}$, where $S_1 = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$.

Chord of Contact

- For $x^2 + y^2 = r^2$ from $P(x_1, y_1)$: $xx_1 + yy_1 = r^2$ (or $T=0$).
- For $x^2 + y^2 + 2gx + 2fy + c = 0$ from $P(x_1, y_1)$:
 $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

Angle Between Two Tangents

- Angle between two tangents from P to $x^2 + y^2 = r^2$: $\angle ACB = 2 \tan^{-1}(r/\sqrt{S})$, where S is the power of P .

Director Circle

- Locus of intersection of two perpendicular tangents to $x^2 + y^2 = r^2$: $x^2 + y^2 = 2r^2$.

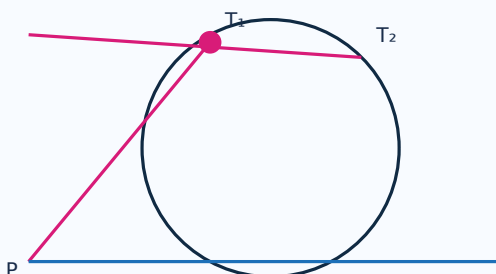
Pole & Polar

- Polar of $P(x_1, y_1)$ w.r.t. $x^2 + y^2 = r^2$: $xx_1 + yy_1 = r^2$.
- Polar of $P(x_1, y_1)$ w.r.t. $x^2 + y^2 + 2gx + 2fy + c = 0$:
 $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.
- Two points are conjugate if each lies on the polar of the other.

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$S S_1 = T^2$	Chord of contact: $T=0$	Director circle: $x^2 + y^2 = 2r^2$
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PAIR OF TANGENTS / POLAR



$$SS_1 = T^2 \quad \bullet \quad T=0$$

polar / chord of contact

5. Two Circles: Common Tangents & Common Chord

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Common Tangents — Number & Conditions

- Four common tangents: $C_1C_2 > r_1 + r_2$.
- Three common tangents (external touching): $C_1C_2 = r_1 + r_2$.
- Two common tangents (intersecting): $|r_1 - r_2| < C_1C_2 < r_1 + r_2$.
- One common tangent (internal touching): $C_1C_2 = |r_1 - r_2|$.
- No common tangent (one circle inside another): $C_1C_2 < |r_1 - r_2|$.

Length of Common Tangents

- Direct common tangent: $\sqrt{C_1C_2^2 - (r_1 - r_2)^2}$.
- Transverse common tangent: $\sqrt{C_1C_2^2 - (r_1 + r_2)^2}$.

Common Chord & Radical Axis

- For $S_1=0$ and $S_2=0$, common chord: $S_1 - S_2 = 0$.
- The radical axis is perpendicular to the line joining the centres.
- Radical axes of three circles with non-collinear centres are concurrent at the radical centre.

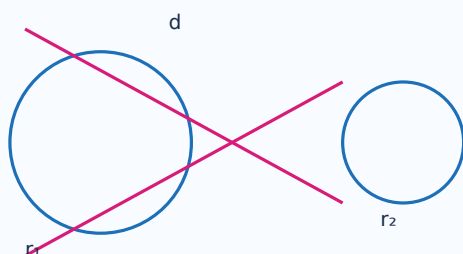
Angle of Intersection

- For two circles with radii r_1, r_2 and centre distance d : $\cos \theta = \frac{(r_1^2 + r_2^2 - d^2)}{(2r_1r_2)}$.

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4 / 3 / 2 / 1 / 0 tangents	Direct: $\sqrt{d^2 - (r_1 - r_2)^2}$	Transverse: $\sqrt{d^2 - (r_1 + r_2)^2}$
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TWO CIRCLES



Compare d with $r_1 + r_2$

direct & transverse tangents

6. Orthogonal Circles, Family of Circles & Radical Axis

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Orthogonal Circles

- If $S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ are orthogonal: $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.
- For two orthogonally intersecting circles, common chord length = $2r_1r_2/\sqrt{r_1^2 + r_2^2}$.

Radical Axis

- For $S_1 = 0$ and $S_2 = 0$, radical axis: $S_1 - S_2 = 0$.
- A point on the radical axis has equal tangent lengths to the two circles.
- The centre of a circle cutting two given circles orthogonally lies on their radical axis.

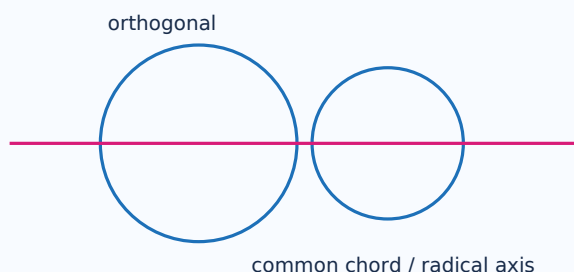
Family of Circles

- Through intersection of circle $S = 0$ and line $L = 0$: $S + \lambda L = 0$.
- Through intersection of two circles $S_1 = 0$ and $S_2 = 0$: $S_1 + \lambda S_2 = 0$ ($\lambda \neq -1$).
- Family touching $S = 0$ at $P(x_1, y_1)$: $S + \lambda L = 0$, where $L = 0$ is tangent at P .
- Family through points $A(x_1, y_1)$ and $B(x_2, y_2)$: use the two-point family form given in the handbook.

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Orthogonal: $2g_1g_2 + 2f_1f_2 = c_1 + c_2$	Family: $S + \lambda L = 0$	Radical axis: $S_1 - S_2 = 0$
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ORTHOGONAL CIRCLES



$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

right-angle intersection

7. Coaxial System, Limiting Points & Special Results

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Coaxial System

- A coaxial system is a system in which every pair of circles has the same radical axis.
- If common radical axis is $P \equiv lx + my + n = 0$ and one circle is $S = 0$, the coaxial system is $S + \lambda P = 0$.
- Centres of all circles of a coaxial system lie on a straight line perpendicular to the common radical axis.

Limiting Points

- For the coaxial system $x^2 + y^2 + 2gx + c = 0$, $\text{radius}^2 = g^2 - c$.
- At a limiting point $r = 0 \Rightarrow g^2 - c = 0 \Rightarrow g = \pm\sqrt{c}$.
- Limiting points are $(\sqrt{c}, 0)$ and $(-\sqrt{c}, 0)$ for the stated system.

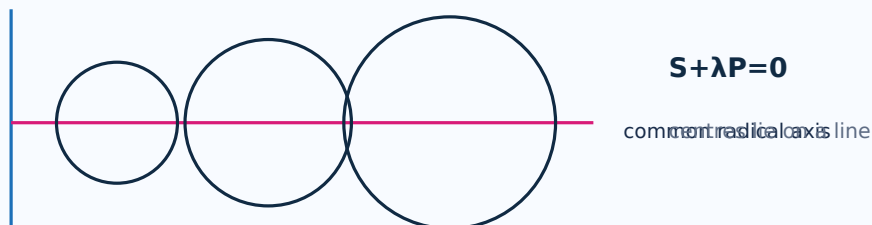
Important Special Results

- Pole of $lx + my + n = 0$ w.r.t. $x^2 + y^2 = a^2$: $(-a^2l/n, -a^2m/n)$.
- For two circles $S_1 = 0$, $S_2 = 0$ with radii r_1, r_2 , the circle $S_1/r_1 \pm S_2/r_2 = 0$ meets them at right angle.
- Family of circles touching a line $L = 0$ at (x_1, y_1) : $(x - x_1)^2 + (y - y_1)^2 + \lambda L = 0$.
- Diameter bisecting parallel chords $y = mx + c$ of $x^2 + y^2 = a^2$: $x + my = 0$.
- A diameter of a circle passes through its centre and is perpendicular to the corresponding family of parallel chords.

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Coaxial: $S + \lambda P = 0$	Limiting points: $(\pm\sqrt{c}, 0)$	Parallel-chord diameter: $x + my = 0$
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COAXIAL SYSTEM



8. One-Page JEE Main Quick Revision Map

A last-minute scan sheet — formulas to recall before solving PYQs.

Core equation	$(x-h)^2+(y-k)^2=r^2$
General equation	$x^2+y^2+2gx+2fy+c=0$; $C(-g,-f)$, $R=\sqrt{g^2+f^2-c}$
Point test	$S>0$ outside, $S=0$ on, $S<0$ inside
Tangent at P	$T=0$
Normal	Perpendicular to tangent and passes through centre
Chord of contact	$T=0$ for the external point
Director circle	$x^2+y^2=2r^2$ for $x^2+y^2=r^2$
Radical axis	$S_1-S_2=0$
Orthogonal circles	$2g_1g_2+2f_1f_2=c_1+c_2$
Family	$S+\lambda L=0$ or $S_1+\lambda S_2=0$
Common chord (orthogonal)	$2r_1r_2/\sqrt{(r_1^2+r_2^2)}$
Diameter of parallel chords	For $y=mx+c$: $x+my=0$

Study Strategy

- Memorise the standard/general forms first, then practise $T=0$ based tangent/chord-of-contact questions.
- For two-circle questions, identify centre distance d and compare it with r_1+r_2 and $|r_1-r_2|$.
- Use $S_1-S_2=0$ as a fast route for common chord and radical-axis questions.
- Verify the geometric condition before applying a formula.

CIRCLE PROBLEM FLOW

1 Identify the circle	2 Write centre & radius	3 Check geometry / condition	4 Apply the shortest formula
01 Know the form Standard $\leftrightarrow$ General equation	02 Check the geometry Distance, tangent, intersection	03 Use the shortcut $T=0$ • $S_1-S_2=0$ • $S+\lambda L=0$	

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