



GURU HOME TUTORS

JEE MAIN • MATHEMATICS • PRACTICE MATERIAL

JEE MAIN PRACTICE WORKBOOK

PARABOLA

Concept-based questions with detailed solutions

STANDARD FORM

$$y^2 = 4ax$$

FOCUS

$$(a, 0)$$

LATUS RECTUM

$$4a$$

Prepared by Prasum Mishra

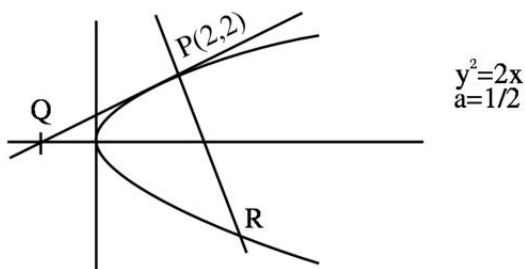
PARABOLA

- Let the tangent to the parabola $S : y^2 = 2x$ at the point $P(2, 2)$ meet the x -axis at Q and normal at it meet the parabola S at the point R . Then the area (in sq. units) of the triangle PQR is equal to:
 (1) $\frac{25}{2}$ (2) $\frac{35}{2}$ (3) $\frac{15}{2}$ (4) 25
- Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m + c)$ is equal to _____.
- Let P be a variable point on the parabola $y = 4x^2 + 1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line $y = x$ is :
 (1) $(3x - y)^2 + (x - 3y) + 2 = 0$
 (2) $2(3x - y)^2 + (x - 3y) + 2 = 0$
 (3) $(3x - y)^2 + 2(x - 3y) + 2 = 0$
 (4) $2(x - 3y)^2 + (3x - y) + 2 = 0$
- If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β) , then $2(\alpha + \beta)$ is equal to _____.
- Let a parabola P be such that its vertex and focus lie on the positive x -axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from $O(0, 0)$ to the parabola P which meet P at S and R , then the area (in sq. units) of ΔSOR is equal to :
 (1) $16\sqrt{2}$ (2) 16 (3) 32 (4) $8\sqrt{2}$
 If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$, then the length of this chord is :
 (1) 5 (2) 7 (3) $5\sqrt{3}$ (4) $3\sqrt{5}$
- The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is :
 (1) $y^3(x - 2) = x^2$ (2) $x^3(x - 2) = y^2$
 (3) $y^2(x - 2) = x^3$ (4) $x^2(x - 2) = y^3$
- A tangent and a normal are drawn at the point $P(2, -4)$ on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If $Q(a, b)$ is a point such that $AQBP$ is a square, then $2a + b$ is equal to :
 (1) -16 (2) -18 (3) -12 (4) -20
- If two tangents drawn from a point P to the parabola $y^2 = 16(x - 3)$ are at right angles, then the locus of point P is :
 (1) $x + 3 = 0$ (2) $x + 1 = 0$
 (3) $x + 2 = 0$ (4) $x + 4 = 0$
- The length of the latus rectum of a parabola, whose vertex and focus are on the positive x -axis at a distance R and S ($>R$) respectively from the origin, is :
 (1) $4(S + R)$ (2) $2(S - R)$
 (3) $4(S - R)$ (4) $2(S + R)$
- A tangent line L is drawn at the point $(2, -4)$ on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to _____.
- Consider the parabola with vertex $\left(\frac{1}{2}, \frac{3}{4}\right)$, the directrix $y = \frac{1}{2}$. Let P be the point where parabola meets the line $x = -\frac{1}{2}$. If the normal to the parabola at P intersects the parabola again at the point Q , then $(PQ)^2$ is equal to :
 (1) $\frac{75}{8}$ (2) $\frac{125}{16}$ (3) $\frac{25}{2}$ (4) $\frac{15}{2}$

13. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are :
(1) (3, 13) (2) (1, 5) (3) (-2, 8) (4) (2, 8)
14. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is :
(1) $x = -\frac{a}{2}$ (2) $x = \frac{a}{2}$
(3) $x = 0$ (4) $x = a$
15. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it?
(1) (-6, 0) (2) (4, 5) (3) (5, 4) (4) (0, 3)
16. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is :
(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) 0
17. A line is a common tangent to the parabola $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a + c)$ is equal to _____.
18. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at P(2,1) is :
(1) $x - y = 1$ (2) $2x + y = 5$
(3) $x + 3y = 5$ (4) $x + 2y = 4$
19. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point (a, 0) $a \neq 0$, then 'a' must be greater than :
(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) -1 (4) 1

1. Official Ans. by NTA (1)

Sol.



Tangent at P : $y(2) = 2(1/2)(x + 2)$

$$\Rightarrow 2y = x + 2$$

$$\therefore Q = (-2, 0)$$

Normal at P : $y - 2 = -\frac{(2)}{2 \cdot \frac{1}{2}}(x - 2)$

$$\Rightarrow y - 2 = -2(x - 2)$$

$$\Rightarrow y = 6 - 2x$$

$$\therefore \text{Solving with } y^2 = 2x \Rightarrow R\left(\frac{9}{2} - 3\right)$$

$$\therefore \text{Ar}(\Delta PQR) = \frac{1}{2} \begin{vmatrix} 2 & 2 & 1 \\ -2 & 1 & 1 \\ \frac{9}{2} & 3 & 1 \end{vmatrix}$$

$$= \frac{25}{2} \text{ sq. units}$$

Official Ans. by NTA (34)

l. $y^2 = -64x$

focus : $(-16, 0)$

$y = mx + c$ is focal chord

$$\Rightarrow c = 16m \dots\dots(1)$$

$y = mx + c$ is tangent to $(x + 10)^2 + y^2 = 4$

$$\Rightarrow y = m(x + 10) \pm 2\sqrt{1 + m^2}$$

$$\Rightarrow c = 10m \pm 2\sqrt{1 + m^2}$$

$$\Rightarrow 16m = 10m \pm 2\sqrt{1 + m^2}$$

$$\Rightarrow 6m = 2\sqrt{1 + m^2} \quad (m > 0)$$

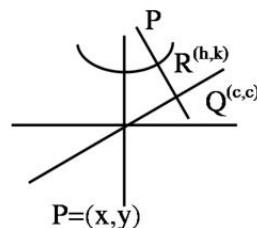
$$\Rightarrow 9m^2 = 1 + m^2$$

$$\Rightarrow m = \frac{1}{2\sqrt{2}} \text{ \& } c = \frac{8}{\sqrt{2}}$$

$$4\sqrt{2}(m + c) = 4\sqrt{2}\left(\frac{17}{2\sqrt{2}}\right) = \boxed{34}$$

3. Official Ans. by NTA (2)

Sol.



$$\frac{K - C}{h - C} = -1$$

$$C = \frac{h + K}{2} \quad P(x, y)$$

$$R = \left(\frac{x + C}{2}, \frac{y + C}{2}\right)$$

$$R = \left(\frac{x}{2} + \frac{h}{4} + \frac{K}{4}, \frac{y}{2} + \frac{h}{4} + \frac{k}{4}\right)$$

$$h = \frac{x}{2} + \frac{h}{4} + \frac{K}{4}$$

$$K = \frac{y}{2} + \frac{h}{4} + \frac{K}{4}$$

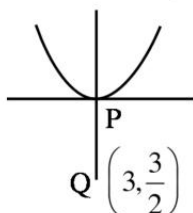
$$\Rightarrow x = \frac{3h}{2} - \frac{K}{2}, y = \frac{3K}{2} - \frac{h}{2}$$

$$Y = 4x^2 + 1$$

$$\left(\frac{3k - h}{2}\right) = 4\left(\frac{3h - k}{2}\right)^2 + 1$$

4. Official Ans. by NTA (9)

Sol.



$$P \equiv \left(\frac{3}{2}t^2, 3t \right)$$

Normal at point P

$$tx + y = 3t + \frac{3}{2}t^3$$

Passes through $\left(3, \frac{3}{2} \right)$

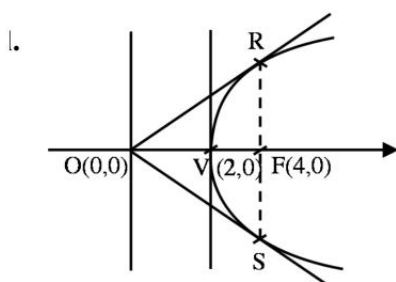
$$\Rightarrow 3t + \frac{3}{2} = 3t + \frac{3}{2}t^3$$

$$P \equiv \left(\frac{3}{2}, 3 \right) = (\alpha, \beta)$$

$$\Rightarrow t^3 = 1 \Rightarrow t = 1$$

$$2(\alpha + \beta) = 2\left(\frac{3}{2} + 3 \right) = 9$$

5. Official Ans. by NTA (2)



Clearly RS is latus-rectum

$$\therefore VF = 2 = a$$

$$\therefore RS = 4a = 8$$

$$\text{Now } OF = 2a = 4$$

$$\Rightarrow \text{Area of triangle ORS} = 16$$

6. Official Ans. by NTA (4)

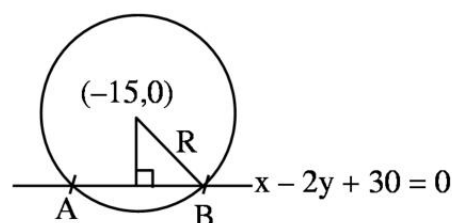
Sol. Equation of tangent to $y^2 = 30x$

$$y = mx + \frac{30}{4m}$$

$$\text{Pass thru } (-30, 0) : a = -30m + \frac{30}{4m} \Rightarrow m^2 = 1/4$$

$$\Rightarrow m = \frac{1}{2} \text{ or } m = -\frac{1}{2}$$

$$\text{At } m = \frac{1}{2} : y = \frac{x}{2} + 15 \Rightarrow x - 2y + 30 = 0$$



$$P = \frac{15}{\sqrt{5}}$$

$$\ell_{AB} = 2\sqrt{R^2 - P^2} = 2\sqrt{\frac{225}{4} - \frac{225}{5}}$$

$$\Rightarrow \ell_{AB} = 30 \cdot \sqrt{\frac{1}{20}} = \frac{15}{\sqrt{5}} = 3\sqrt{5}$$

7. Official Ans. by NTA (3)

Sol. $T = S_1$

$$xh - yk = h^2 - k^2$$

$$y = \frac{xh}{k} - \frac{(h^2 - k^2)}{k}$$

$$\text{this touches } y^2 = 8x \text{ then } c = \frac{a}{m}$$

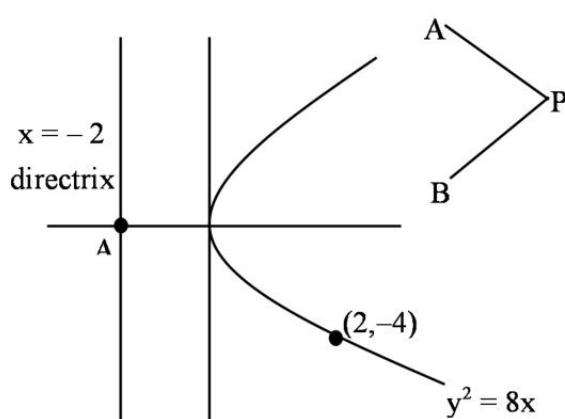
$$\left(\frac{k^2 - h^2}{k} \right) = \frac{2k}{h}$$

$$2y^2 = x(y^2 - x^2)$$

$$y^2(x - 2) = x^3$$

y NTA (1)

Sol.



Equation of tangent at (2, -4) (T = 0)

$$-4y = 4(x + 2)$$

$$x + y + 2 = 0 \quad \dots(1)$$

equation of normal

$$x - y + \lambda = 0$$

$$\downarrow (2, -4)$$

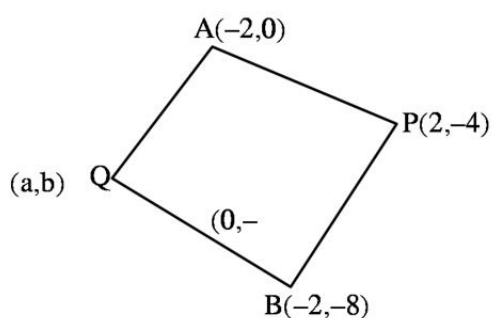
$$\lambda = -6$$

thus $x - y = 6 \dots(2)$ equation of normal

POI of (1) & $x = -2$ is A(-2, 0)

POI of (2) & $x = -2$ is A(-2, 8)

Given AQBP is a sq.



$$\Rightarrow m_{AQ} \cdot m_{AP} = -1$$

$$\Rightarrow \left(\frac{b}{a+2} \right) \left(\frac{4}{-4} \right) = -1 \Rightarrow a+2 = b \quad \dots(1)$$

Also PQ must be parallel to x-axis thus

$$\Rightarrow b = -4$$

$$\therefore a = -6$$

$$\text{Thus } 2a + b = -16$$

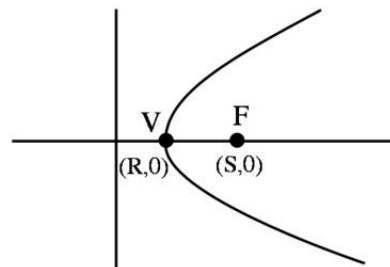
9. Official Ans. by NTA (2)

Sol. Locus is directrix of parabola

$$x - 3 + 4 = 0 \Rightarrow x + 1 = 0.$$

10. Official Ans. by NTA (3)

Sol.


V $\rightarrow$ Vertex

F $\rightarrow$ focus

$$VF = S - R$$

$$\text{So latus rectum} = 4(S - R)$$

11. Official Ans. by NTA (2)

Sol. tangent of $y^2 = 8x$ is $y = mx + \frac{2}{m}$

$$P(2, -4) \Rightarrow -4 = 2m + \frac{2}{m}$$

$$\Rightarrow m + \frac{1}{m} = -2 \Rightarrow m = -1$$

$$\therefore \text{tangent is } y = -x - 2$$

$$\Rightarrow x + y + 2 = 0 \quad \dots(1)$$

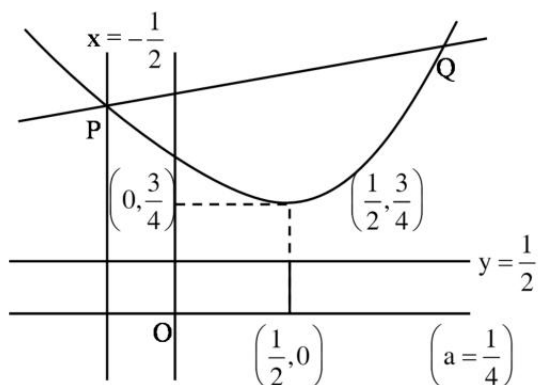
(1) is also tangent to $x^2 + y^2 = a$

$$\text{So } \frac{2}{\sqrt{2}} = \sqrt{a} \Rightarrow \sqrt{a} = \sqrt{2}$$

$$\Rightarrow a = 2$$

12. Official Ans. by NTA (2)

Sol.



$$\left(y - \frac{3}{4}\right) = \left(x - \frac{1}{2}\right)^2 \dots (1)$$

$$\text{For } x = -\frac{1}{2}$$

$$y - \frac{3}{4} = 1 \Rightarrow y = \frac{7}{4} \Rightarrow P\left(-\frac{1}{2}, \frac{7}{4}\right)$$

$$\text{Now } y' = 2\left(x - \frac{1}{2}\right) \quad \text{At } x = -\frac{1}{2}$$

$$\Rightarrow m_T = -2, m_N = \frac{1}{2}$$

Equation of Normal is

$$y - \frac{7}{4} = \frac{1}{2}\left(x + \frac{1}{2}\right)$$

$$y = \frac{x}{2} + 2$$

Now put y in equation (1)

$$\frac{x}{2} + 2 - \frac{3}{4} = \left(x - \frac{1}{2}\right)^2$$

$$\Rightarrow x = 2 \text{ \& } -\frac{1}{2}$$

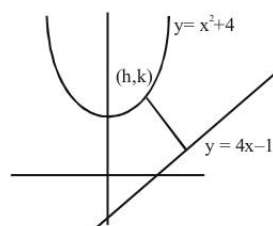
$$\Rightarrow Q(2, 3)$$

$$\text{Now } (PQ)^2 = \frac{125}{16}$$

Option (2)

13. Official Ans. by NTA (4)

Sol. Ans. (4)



$$P : y = x^2 + 4$$

$$k = h^2 + 4$$

$$L : y = 4x - 1$$

$$y - 4x + 1 = 0$$

$$d = AB = \left| \frac{k - 4h + 1}{\sqrt{5}} \right| = \left| \frac{h^2 - 4 - 4h + 1}{\sqrt{5}} \right|$$

$$\frac{d(d)}{dh} = \frac{2h - 4}{\sqrt{5}} = 0$$

$$h = 2$$

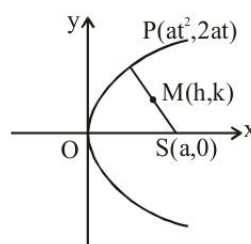
$$\frac{d^2(d)}{dh^2} = \frac{2}{\sqrt{5}} > 0$$

$$\therefore k = 4 + 4 = 8$$

$$\therefore \text{Point } (2, 8)$$

14. Official Ans. by NTA (3)

Sol.



$$h = \frac{at^2 + a}{2}, k = \frac{2at + 0}{2}$$

$$\Rightarrow t^2 = \frac{2h - a}{a} \text{ and } t = \frac{k}{a}$$

$$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$$

$$\Rightarrow \text{Locus of } (h, k) \text{ is } y^2 = a(2x - a)$$

$$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$$

$$\text{Its directrix is } x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$$

y NTA (3)

Sol. Slope of tangent = $m_T = m$

$$\text{So, } m(-2) = -1 \Rightarrow m = \frac{1}{2}$$

$$\text{Equation : } y = mx + \frac{a}{m}$$

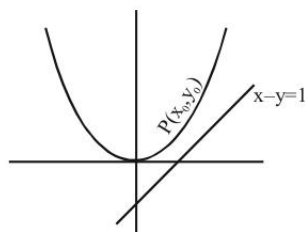
$$\Rightarrow y = \frac{1}{2}x + \frac{3}{2 \times \frac{1}{2}} \left(a = \frac{6}{4} = \frac{3}{2} \right)$$

$$\Rightarrow y = \frac{x}{2} + 3$$

$$\Rightarrow 2y = x + 6$$

Point (5, 4) will not lie on it

16. Official Ans. by NTA (2)



Sol.

Shortest distance between curves is always along common normal.

$$\left. \frac{dy}{dx} \right|_P = \text{slope of line} = 1$$

$$\Rightarrow x_0 = 1 \quad \therefore y_0 = \frac{1}{2}$$

$$\Rightarrow P\left(1, \frac{1}{2}\right)$$

$$\therefore \text{Shortest distance} = \left| \frac{1 - \frac{1}{2} - 1}{\sqrt{1^2 + 1^2}} \right| = \frac{1}{2\sqrt{2}}$$

option (2)

17. Official Ans. by NTA (9)

Sol. Let coordinate of point A(t^2 , $2t$) ($\because a = 1$)

equation of tangent at point A

$$yt = x + t^2$$

$$x - ty + t^2 = 0$$

centre of circle (3, 0)

Now PD = radius

$$\left| \frac{3 - 0 + t^2}{\sqrt{1 + t^2}} \right| = 3$$

$$(3 + t^2)^2 = 9(1 + t^2)$$

$$9 + t^4 + 6t^2 = 9 + 9t^2$$

$$t = 0, -\sqrt{3}, \sqrt{3}$$

So point A($3, 2\sqrt{3}$)

$$\Rightarrow a = 3, b = 2\sqrt{3}$$

(Since it lies in first quadrant)

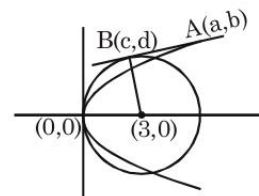
For point B which is foot of perpendicular from

centre (3, 0) to the tangent $x - \sqrt{3}y + 3 = 0$

$$\frac{c - 3}{1} = \frac{d - 0}{-\sqrt{3}} = \frac{-(3 - 0 + 3)}{4}$$

$$\Rightarrow c = \frac{3}{2} \quad d = \frac{3\sqrt{3}}{2}$$

$$\Rightarrow 2\left(\frac{3}{2} + 3\right) = 9$$



18. Official Ans by NTA (1)**Sol.** Given $y^2 = 4x$ Mirror image on $y = x \Rightarrow C : x^2 = 4y$

$$2x = 4 \cdot \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$$

$$\left. \frac{dy}{dx} \right|_{P(2,1)} = \frac{2}{2} = 1$$

Equation of tangent at (2, 1)

$$\Rightarrow y - 1 = 1(x - 2)$$

$$\Rightarrow x - y = 1$$

19. Official Ans. by NTA (4)**Sol.** For standard parabola

For more than 3 normals (on axis)

$$x > \frac{L}{2} \text{ (where L is length of L.R.)}$$

$$\text{For } y^2 = 2x$$

$$\text{L.R.} = 2$$

for (a, 0)

$$a > \frac{\text{L.R.}}{2} \Rightarrow a > 1$$