

QUESTION 01

21 JAN • SHIFT 2

For the parabola $y^2 = 12x$, P is a point on it and A is on the x-axis.

If $\angle OPA = 90^\circ$, find the locus of the centroid of $\triangle OPA$.

A) $y^2 - 2x + 8 = 0$

C) $y^2 - 4x + 8 = 0$

B) $y^2 - 9x + 6 = 0$

D) $y^2 - 6x + 4 = 0$

YOUR WORKING

rough work / alternate method



Q01 • SOLUTION

STEP

For $y^2 = 12x$, $a = 3$.

$$P = (3t^2, 6t), \quad A = (h, 0)$$

STEP

$$\angle OPA = 90^\circ \Rightarrow OP \cdot PA = 0$$

$$(-3t^2)(h - 3t^2) + (-6t)(-6t) = 0$$

STEP

$$h = 3t^2 + 12$$

$$G = ((3t^2+h)/3, 6t/3) = (2t^2+4, 2t)$$

$$y = 2t, \quad x = y^2/2 + 4$$

$$y^2 - 2x + 8 = 0$$

ANSWER

$$y^2 - 2x + 8 = 0$$

QUESTION 02

21 JAN • SHIFT 2

One end of a focal chord of $y^2 = 16x$ is $(16,16)$.

$P(\alpha,\beta)$ divides the focal chord internally in the ratio 5:2. Find the minimum value of $\alpha+\beta$.

A) 16

C) 7

B) 5

D) 22

YOUR WORKING

rough work / alternate method



Q02 • SOLUTION

STEP

For $y^2 = 16x$, $a = 4$. $(16,16)$ corresponds to $t = 2$.

For a focal chord: $t_1 t_2 = -1$

STEP

$t_1 = 2, \quad t_2 = -1/2$

Other endpoint $Q = (1, -4)$

STEP

Internal division in ratio 5 : 2 gives two orientations.

$\alpha + \beta = 7 \quad \text{or} \quad \alpha + \beta = 22$

Minimum $(\alpha + \beta) = 7$

ANSWER

7

QUESTION 03

28 JAN • SHIFT 2

A is the focus of $y^2 = 8x$. The line $y = mx + c$ intersects the parabola at B and C. The centroid of $\triangle ABC$ is $(7/3, 4/3)$. Find $(BC)^2$.

A) 80

C) 89

B) 41

D) 32

YOUR WORKING

rough work / alternate method



Q03 • SOLUTION

STEP

For $y^2 = 8x$, $a = 2$.

$$B = (2t_1^2, 4t_1), \quad C = (2t_2^2, 4t_2), \quad A = (2, 0)$$

STEP

Centroid = $(7/3, 4/3)$ gives $t_1^2 + t_2^2 = 5/2$

$$\text{Also } t_1 + t_2 = 1$$

STEP

$$t_1 t_2 = [(t_1 + t_2)^2 - (t_1^2 + t_2^2)]/2 = -3/4$$

$$(t_1 - t_2)^2 = (t_1 + t_2)^2 - 4t_1 t_2 = 4$$

$$BC^2 = [2(t_1^2 - t_2^2)]^2 + [4(t_1 - t_2)]^2$$

$$= 4(t_1 - t_2)^2(t_1 + t_2)^2 + 16(t_1 - t_2)^2$$

$$BC^2 = 16 + 64 = 80$$

ANSWER
80

QUESTION 04

24 JAN • SHIFT 2

The image of $x^2 = 4y$ in the line $x - y = 1$ is $(y+a)^2 = b(x-c)$, where $a, b, c \in \mathbb{N}$. Find $a+b+c$.

A) 8**C) 12****B) 6****D) 4**

YOUR WORKING

rough work / alternate method



Q04 • SOLUTION

STEP

Reflect $x^2 = 4y$ in $x - y = 1$.

$$x' = y + 1, \quad y' = x - 1$$

STEP

$$x = y' + 1, \quad y = x' - 1$$

$$(y' + 1)^2 = 4(x' - 1)$$

STEP

$$\text{Image: } (y + 1)^2 = 4(x - 1)$$

$$a = 1, \quad b = 4, \quad c = 1$$

$$a + b + c = 6$$

ANSWER

6

QUESTION 05

22 JAN • SHIFT 2

Let S be the locus of the midpoint of a chord through the origin of $y^2 = 4x$.
If P divides OM internally in the ratio 3:1, find the locus of P.

A) $2x^2 = 3y$

C) $2y^2 = 3x$

B) $3y^2 = 2x$

D) $3x^2 = 2y$

YOUR WORKING

rough work / alternate method



Q05 • SOLUTION

STEP

For $y^2 = 4x$, let $Q = (t^2, 2t)$.Midpoint $M = (t^2/2, t)$

STEP

So $S : y^2 = 2x$ **P divides OM internally in ratio 3 : 1.**

STEP

 $P = (3t^2/8, 3t/4)$

$$x = 3t^2/8, \quad y = 3t/4$$

$$2y^2 = 3x$$

ANSWER
 $2y^2 = 3x$

QUESTION 06

22 JAN • SHIFT 1

A chord joining $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on $y^2 = 12x$ subtends a right angle at the vertex. Find $x_1x_2 - y_1y_2$.

A) 284

C) 288

B) 280

D) 292

YOUR WORKING

rough work / alternate method



Q06 • SOLUTION

STEP

For $y^2 = 12x$, $a = 3$.

$$P_i = (3t_i^2, 6t_i), \quad i = 1, 2$$

STEP

Right angle at O means $OP_1 \perp OP_2$.

$$OP_1 \cdot OP_2 = 0$$

STEP

$$9t_1^2t_2^2 + 36t_1t_2 = 0 \Rightarrow t_1t_2 = -4$$

$$x_1x_2 - y_1y_2 = 9t_1^2t_2^2 - 36t_1t_2$$

$$= 9(16) - 36(-4) = 288$$

ANSWER
288

QUESTION 07

23 JAN • SHIFT 2

An equilateral triangle OAB is inscribed in $y^2 = 4x$, with O as a vertex.

Find the minimum distance of the circle with AB as diameter from the origin.

A) $2(8 - 3\sqrt{3})$

C) $4(6 + \sqrt{3})$

B) $2(3 + \sqrt{3})$

D) $4(3 - \sqrt{3})$

YOUR WORKING

rough work / alternate method



Q07 • SOLUTION

STEP

For $y^2 = 4x$, take $A = (t^2, 2t)$, $B = (t^2, -2t)$.

$$AB = 4|t|$$

STEP

$$OA = |t|\sqrt{t^2 + 4}$$

For an equilateral triangle, $AB = OA$.

STEP

$$4|t| = |t|\sqrt{t^2 + 4} \Rightarrow t^2 = 12$$

$$A = (12, 4\sqrt{3}), \quad B = (12, -4\sqrt{3})$$

Circle centre = $(12, 0)$, radius = $4\sqrt{3}$

$$\text{Minimum distance} = 12 - 4\sqrt{3} = 4(3 - \sqrt{3})$$

ANSWER
 $4(3 - \sqrt{3})$

QUESTION 08

21 JAN • SHIFT 1

O is the vertex of $x^2 = 4y$. Q lies on the parabola and P divides OQ internally in the ratio 2:3.

The locus of P is conic C. Find the chord of C bisected at (1,2).

A) $5x - 4y + 3 = 0$

C) $4x - 5y + 6 = 0$

B) $5x - y - 3 = 0$

D) $x - 2y + 3 = 0$

YOUR WORKING

rough work / alternate method



Q08 • SOLUTION

STEP

For $x^2 = 4y$, let $Q = (2t, t^2)$.

$$P = (4t/5, 2t^2/5)$$

STEP

$$x = 4t/5, \quad y = 2t^2/5$$

$$C : x^2 = 8y/5$$

STEP

So C is $x^2 = 4ay$ with $a = 2/5$.

$$\text{Chord with midpoint (h,k): } hx - 2ay = h^2 - 2ak$$

$$h = 1, k = 2 \Rightarrow x - 4y/5 = -3/5$$

$$5x - 4y + 3 = 0$$

ANSWER

$$5x - 4y + 3 = 0$$